

Electron and Phonon Thermal Conduction in Epitaxial High- T_c Superconducting Films

K. E. Goodson

M. I. Flik

Department of Mechanical Engineering,
Massachusetts Institute of Technology,
Cambridge, MA 02139

Electrons and phonons are the carriers of heat in the a - b plane of the high- T_c superconductor $\text{YBa}_2\text{Cu}_3\text{O}_7$. In the absence of boundary scattering, the a - b plane thermal conductivity and the mean free path of each carrier type are calculated as functions of temperature using kinetic theory, the two-fluid model of the superconducting state, and experimental data for the thermal conductivity and electrical resistivity of a single crystal. The reduction by boundary scattering of the effective a - b plane thermal conductivity along an epitaxial $\text{YBa}_2\text{Cu}_3\text{O}_7$ film is predicted as a function of temperature and film thickness. The size effect on the phonon conductivity dominates over the size effect on the electron conductivity. The predicted electron mean free path is limited by scattering on defects and is in very good agreement with experimental data from infrared spectroscopy.

Introduction

In epitaxial high- T_c superconducting films with the crystallographic c axis normal to the plane of the film, conduction along the film occurs in the a - b plane. Electrons are responsible for 30–50 percent of thermal conduction in the a - b plane of single-crystal $\text{YBa}_2\text{Cu}_3\text{O}_7$ near its critical temperature, $T_c = 92$ K (Cohn et al., 1992b; Hagen et al., 1989), and 30–40 percent of thermal conduction in the a - b plane of single crystal $\text{Bi}_2\text{Sr}_2\text{Ca}_1\text{Cu}_2\text{O}_8$ near $T_c = 89$ K (Crommie and Zettl, 1990). Lattice vibrations, the energy quanta of which are phonons, account for the remaining fraction of the thermal conduction. Boundary scattering reduces the effective thermal conductivity along a film when the mean free path of a carrier is of the order of or larger than the film thickness. Determination of the effective conductivity along epitaxial high- T_c superconducting films must consider the mean free paths and contributions to the total conductivity of electrons and phonons in the absence of boundary scattering. At present, none of these quantities is known for temperatures lower than the critical temperature.

Epitaxial superconducting films possess great promise for application in electronic circuits (Van Duzer, 1988). The performance of superconducting devices, such as Josephson junctions, in hybrid electronic circuits is affected by temperature fields, the prediction of which requires the thermal conductivity (Flik and Hijikata, 1990; Lavine and Bai, 1990). The liquid-nitrogen bolometer is an infrared radiation detector that exploits the drastic change of electrical resistivity with temperature of a high- T_c superconducting film at the midpoint of its transition. The film and substrate thermal conductivities are required for the determination of the thermal behavior of this device (Flik et al., 1990; Kumar and Joshi, 1990). The thermal conductivity of superconducting films is required for the analysis of the thermal stability and the voltage response of thin-film microbridges irradiated by an electron-beam (Flik and Tien, 1990a, 1990b; Flik and Goodson, 1992).

Uher (1990) reviewed research on thermal conduction in high- T_c superconductors. Single crystals of high- T_c superconductors are assumed to possess equal thermal conductivities in the a and b directions, and a different conductivity in the

c direction. This assumption is a good approximation for $\text{YBa}_2\text{Cu}_3\text{O}_7$ because of its orthorhombic, nearly tetragonal crystal structure. As depicted in Fig. 1, the electron and phonon contributions to the conductivity are limited by scattering on defects in the lattice, such as point defects, stacking faults, and grain boundaries, as well as electron-phonon scattering. In thin films, the boundaries of the film can increase the rate of scattering.

Flik and Tien (1990c) determined the thermal conductivity as a function of temperature and film thickness of polycrystalline $\text{YBa}_2\text{Cu}_3\text{O}_7$ films, in which phonons dominate the thermal conduction. Their analysis did not consider electron transport and is not valid for the a - b plane of epitaxial films. Calculation of the size effect for this case requires knowledge of the contributions of electrons and phonons to the a - b plane thermal conductivity in the superconducting state. Geilikman and Kresin (1974) obtained theories for thermal conduction in superconductors that are in agreement with experimental data for low-temperature superconductors for the cases of electron conduction limited by defect scattering and phonon conduction limited by electron scattering. Tewordt and Wölkhausen (1989, 1990) modified the theory of Bardeen et al. (1959) for the phonon conductivity to account for the scattering of phonons on the defects in high- T_c superconductors. Measurements of the a - b plane thermal conductivity of single-crystal $\text{YBa}_2\text{Cu}_3\text{O}_7$ were reported by Hagen et al. (1989), Cohn et al. (1992a, 1992b), who calculated the phonon conductivity in the superconducting state using the theory of Tewordt and Wölkhausen (1990), and Peacor et al. (1991), who measured the influence of magnetic field strength on the thermal conductivity.

This work develops a new technique for calculating the a - b plane thermal conductivity of epitaxial high- T_c supercon-

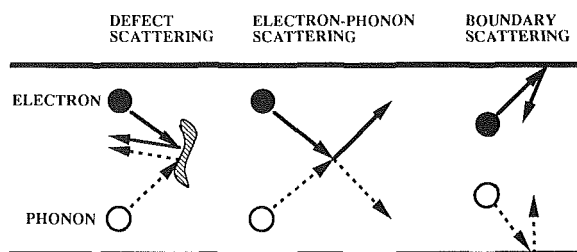


Fig. 1 Scattering mechanisms that limit thermal conduction in high- T_c superconducting films

Contributed by the Heat Transfer Division for publication in the JOURNAL OF HEAT TRANSFER. Manuscript received by the Heat Transfer Division June 1991; revision received April 1992. Keywords: Conduction, Cryogenics, Thermophysical Properties. Associate Technical Editor: L. S. Fletcher.

ducting films from measurements performed on a crystal in which boundary scattering is negligible. The technique is illustrated in the flowchart in Fig. 2. In the absence of boundary scattering, the mean free path of normal-state electrons below the critical temperature is calculated from normal-state electrical resistivity data. The electron thermal conductivity in the absence of boundary scattering in the superconducting state is calculated from the electron mean free path using kinetic theory and the two-fluid model for electrons in superconductors (Gorter and Casimir, 1934; Shoenberg, 1952). The phonon conductivity is obtained by subtracting the electron contribution from measured data in a large crystal, and the phonon mean free path is determined using kinetic theory. For a thin film, the effect of boundary scattering is analyzed separately for the electron and phonon contributions using the approximate relations of Flik and Tien (1990c). The technique is applied to epitaxial $\text{YBa}_2\text{Cu}_3\text{O}_7$ films using the a - b plane electrical resistivity and thermal conductivity data of Cohn et al. (1992b) for a single crystal. The effective thermal conductivity calculated in this work is not a material parameter. It is a geometry-dependent constant, the multiplication of which with the temperature gradient in the a - b plane of the film yields the thermal energy flux in the direction of this gradient.

Electron Thermal Conductivity

This section determines the electron mean free path and thermal conductivity in the absence of boundary scattering. The effect of boundary scattering is considered in the section entitled "Size Effect."

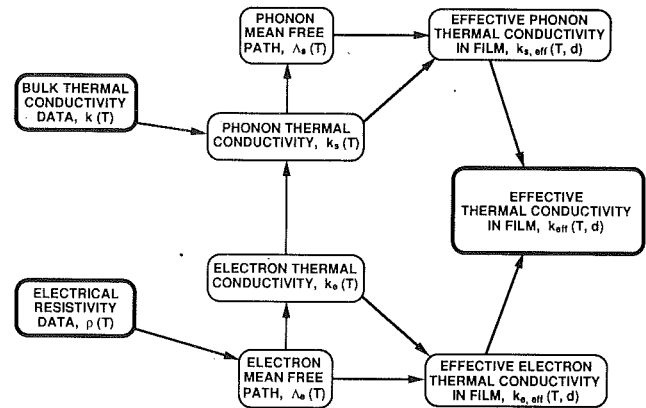


Fig. 2 Flowchart of the technique for determining the effective thermal conductivity along epitaxial high- T_c superconducting films

The mean free path of a carrier Λ is the average distance it travels between consecutive collisions. Matthiessen's rule (Ziman, 1960) relates the electron mean free path Λ_e to the electron mean free path limited by scattering on phonons, $\Lambda_{e,s}$, defects, $\Lambda_{e,d}$, electrons $\Lambda_{e,e}$, and boundaries, $\Lambda_{e,b}$,

$$\frac{1}{\Lambda_e} = \frac{1}{\Lambda_{e,s}} + \frac{1}{\Lambda_{e,d}} + \frac{1}{\Lambda_{e,e}} + \frac{1}{\Lambda_{e,b}} \quad (1)$$

The mean free paths for individual scattering mechanisms are unobservable. They describe the hypothetical situation where

Nomenclature

A = scattering cross section, m^2
 C = specific heat per unit volume, $\text{J m}^{-3} \text{K}^{-1}$
 d = film thickness, m
 E_F = Fermi energy, J
 e = electron charge = $-1.602 \times 10^{-19} \text{C}$
 h_P = Planck's constant = $6.626 \times 10^{-34} \text{J s}$
 J_E = energy flux, W m^{-2}
 $J_p(q)$ = Debye integral, defined by Eq. (6)
 J_S = entropy flux, $\text{W m}^{-2} \text{K}^{-1}$
 k = thermal conductivity, $\text{W m}^{-1} \text{K}^{-1}$
 k_B = Boltzmann constant = $1.381 \times 10^{-23} \text{J K}^{-1}$
 L = exchange length, m
 L_0 = Lorenz number = $2.45 \times 10^{-8} \text{W } \Omega \text{K}^{-2}$
 m^* = effective electron mass, kg
 m = electron rest mass = $9.110 \times 10^{-31} \text{kg}$
 N_a = number density of atoms, m^{-3}
 n = free-electron number density, m^{-3}
 p = argument, Eqs. (6) and (12)
 q, q' = arguments, Eq. (6)
 r = distance from electron, m
 r_B = Bohr radius = $5.292 \times 10^{-11} \text{m}$
 r_s = material parameter, m
 S = electron entropy per unit volume, $\text{J m}^{-3} \text{K}^{-1}$

S_n = normal metal electron entropy per unit volume, $\text{J m}^{-3} \text{K}^{-1}$
 T = temperature, K
 T_c = superconductor critical temperature, K
 T_R = reference temperature, Eq. (7), K
 T_0 = temperature at $y = 0$, K
 v = velocity, m s^{-1}
 X = fraction of free electrons in the normal state
 x = coordinate normal to flow of thermal energy, m
 y = coordinate parallel to flow of thermal energy, m
 Z = parameter, Eq. (24)
 z = coordinate normal to flow of thermal energy, m
 γ = Sommerfeld parameter, $\text{J m}^{-3} \text{K}^{-2}$
 δ = reduced film thickness = d/Λ
 ζ = function, defined in Eq. (5)
 η = coefficient, Eqs. (17) and (18)
 θ = Debye temperature, K
 Λ = mean free path for thermal conduction, m
 ξ = superconductor coherence length, m
 ρ = electrical resistivity, Ωm
 ρ_0 = constant, Eqs. (5) and (8), Ωm

τ = scattering time, s
 Φ = carrier flux through reference plane from one side, $\text{m}^{-2} \text{s}^{-1}$
 χ = function, defined in Eq. (8)
 $d\Omega$ = differential solid angle

Superscripts

$+$ = property of carriers traveling in positive y direction
 $-$ = property of carriers traveling in negative y direction

Subscripts

a - b = property for transport in crystallographic a - b plane
 b = boundary
 c = property for transport parallel to crystallographic c axis
 d = defect
 e = electron
 e, b = electron property limited by boundary scattering
 e, d = electron property limited by defect scattering
 e, e = electron property limited by electron scattering
 e, s = electron property limited by phonon scattering
 eff = effective conductivity reduced by boundary scattering
 l = longitudinal
 s = phonon
 t = transverse

all other scattering mechanisms are inactive. In this section, thermal conduction is examined in the absence of boundary scattering, and the last term on the right is zero. Later in this section, it will be shown that the third term on the right is negligible compared to the second, leaving only the defect and phonon-limited terms.

The mean free path of electrons limited by defect scattering $\Lambda_{e,d}$ is independent of temperature. In a normal metal, this mean free path is related to the residual electrical resistivity ρ_d , the electrical resistivity in the limit of zero temperature, by (Ziman, 1960)

$$\Lambda_{e,d} = \frac{v_e m^*}{\rho_d n e^2}, \quad (2)$$

where v_e is the electron velocity, m^* is the effective electron mass, n is the free-electron number density per unit volume, and e is the electron charge.

Fiory et al. (1990) calculated m^* and n for transport in the a - b plane of $\text{YBa}_2\text{Cu}_3\text{O}_7$ from measurements of the effect of electrostatic charge density modulation on the inductance of an epitaxial $\text{YBa}_2\text{Cu}_3\text{O}_7$ film. This technique provides the most accurate measurements of these properties presently available and was applied to films with thicknesses from 300 to 1000 Å. These film thicknesses are much larger than the mean free path of electrons in $\text{YBa}_2\text{Cu}_3\text{O}_7$, which are calculated later in the present analysis. The data reported for the sample with the lowest normal-state electrical resistivity are employed in the present analysis, $m^* = 4.3 \times 10^{-30}$ kg and $n = 7.0 \times 10^{27} \text{ m}^{-3}$.

A net flux of charge or energy through a plane is carried by electrons possessing energy very near the Fermi energy, E_F . In a Fermi free-electron gas, the Fermi energy is the chemical potential at zero temperature in the Fermi-Dirac energy distribution function (Kittel, 1986). The velocity of electrons possessing the Fermi energy is the Fermi velocity. The Fermi velocity is (Ziman, 1960)

$$v_e = \frac{h_p (3\pi^2 n)^{1/3}}{2\pi m^*}, \quad (3)$$

where h_p is Planck's constant. The Fermi velocity is $v_e = 1.5 \times 10^5 \text{ m s}^{-1}$ for the a - b plane of $\text{YBa}_2\text{Cu}_3\text{O}_7$. The Fermi energy, $E_F = m^* v_e^2 / 2$, is $4.5 \times 10^{-20} \text{ J}$.

The residual electrical resistivity in Eq. (2) cannot be measured directly in a superconductor because a superconductor is not electrically resistive below its critical temperature. The residual resistivity is deduced from the experimentally measured electrical resistivity at higher temperatures using Matthiessen's rule,

$$\rho = \rho_s + \rho_d, \quad (4)$$

where ρ_s is the electrical resistivity resulting from phonons. The electrical resistivity resulting from phonons is determined using the formula of Bloch (1930),

$$\rho_s = 4\rho_\theta \left(\frac{T}{\theta}\right)^5 J_5\left(\frac{T}{\theta}\right) \equiv 4\rho_\theta \zeta\left(\frac{T}{\theta}\right), \quad (5)$$

where θ is the Debye temperature, ρ_θ is a constant that depends on the material, and ζ is a function defined here. The Debye temperature $\theta = 470 \text{ K}$ of $\text{YBa}_2\text{Cu}_3\text{O}_7$ is determined from experimental specific-heat data in the following section. The class of Debye integrals $J_p(q)$ is defined by

$$J_p(q) = \int_0^q \frac{(q')^p e^{q'}}{(e^{q'} - 1)^2} dq'. \quad (6)$$

Equation (5) was derived by neglecting electron-phonon umklapp processes and assuming that the distribution of phonons that scatter electrons is equal to the thermal equilibrium

distribution of phonons in the Debye model of lattice vibrations. Electron-phonon umklapp processes do not conserve the vectorial sum of the electron and phonon momenta. The Bloch formula is in excellent agreement with experimental data for normal metals. The normal-state a - b plane electrical resistivity of $\text{YBa}_2\text{Cu}_3\text{O}_7$ epitaxial films and single crystals varies almost linearly with temperature (Penney et al., 1988; Stormer et al., 1988), which is accommodated within a few percent by Eqs. (4) and (5). Equations (4) and (5) yield the residual resistivity from the total resistivity and its temperature derivative at a reference temperature T_R ,

$$\rho_d = \rho(T_R) - \left[\frac{\zeta\left(\frac{T}{\theta}\right)}{\frac{\partial}{\partial T} \left[\zeta\left(\frac{T}{\theta}\right) \right]} \right]_{T=T_R} \times \left[\frac{\partial \rho}{\partial T} \right]_{T=T_R}. \quad (7)$$

When evaluated with $\theta = 470 \text{ K}$, the first quantity in brackets is equal to 44.0 K at $T_R = 110 \text{ K}$, and 238.1 K at $T_R = 300 \text{ K}$. Using the electrical resistivity measured by Cohn et al. (1992b) for specimen CR3, the residual electrical resistivity is $\rho_d = 0.57 \mu\Omega\text{m}$ and the electron mean free path limited by defect scattering given by Eq. (2) is $\Lambda_{e,d} = 63 \text{ Å}$.

The electron mean free path limited by scattering on electrons, the inverse of the third term on the right of Eq. (1), is estimated by considering the decay of the electrostatic potential due to a single electron in an electron gas (Kittel, 1986). In a metal containing free electrons, the electrostatic potential due to a single electron is proportional to $\exp(-r/r_s)$, where r is the distance from the electron, $r_s = (r_B/4)^{1/2} n^{-1/6}$ is a length that depends on the material, and $r_B = 5.30 \times 10^{-11} \text{ m}$ is the Bohr radius. The electron mean free path limited by scattering on electrons is approximately $\Lambda_{e,e} = (nA)^{-1}$, where A is the scattering cross section for electron-electron collisions. The scattering cross section is approximately the product of the area of a circle of radius r_s and the factor $(k_B T/E_F)^2$, $A \sim (k_B T/E_F)^2 \pi r_s^2$, where k_B is the Boltzmann constant. Combining these relations, $\Lambda_{e,e}$ is approximately $(E_F/k_B T)^2 (r_B n^{2/3})^{-1}$. At a temperature of 90 K, $\Lambda_{e,e}$ in the a - b plane of $\text{YBa}_2\text{Cu}_3\text{O}_7$ is almost three orders of magnitude larger than $\Lambda_{e,d}$ calculated from the electrical resistivity as described above, and contributes negligibly to the sum on the right of Eq. (1). Therefore, electron-electron collisions are neglected in the present analysis.

The electron thermal conductivity limited by phonon scattering in a normal metal is predicted by (Wilson, 1953)

$$\begin{aligned} \left(\frac{1}{k_{e,s}} \right)_{\text{normal}} &= \frac{4\rho_\theta}{L_0 T} \left[\left(\frac{T}{\theta} \right)^5 \left(J_5\left(\frac{\theta}{T}\right) - \frac{1}{2\pi^2} J_7\left(\frac{\theta}{T}\right) \right) \right] \\ &+ \frac{4\rho_\theta}{L_0 T} \left[\left(\frac{T}{\theta} \right)^3 \frac{3}{\pi^2} \left(\frac{n}{2N_a} \right)^{2/3} J_5\left(\frac{\theta}{T}\right) \right] \\ &\equiv \frac{4\rho_\theta}{L_0 T} \chi\left(\frac{T}{\theta}, n, N_a\right), \quad (8) \end{aligned}$$

where L_0 is the Lorenz number, ρ_θ is the constant of Eq. (5), $N_a = 7.5 \times 10^{28} \text{ m}^{-3}$ is the number density of atoms in $\text{YBa}_2\text{Cu}_3\text{O}_7$, and χ is a function defined here. Equation (8) is based on the same assumptions as Eq. (5). In the present analysis, Eq. (8) is approximated by omitting the first bracketed term. This simplification results in an error of less than 34 percent in the ratio $\Lambda_{e,s}(T)/\Lambda_{e,s}(T_c)$, which is employed in the present analysis. The error approaches 34 percent only at the lowest temperatures, where the importance of electron-phonon scattering is negligible compared to that of electron-defect scattering, limiting the effect of this error on the electron conductivity. Kinetic theory yields an expression for the electron thermal conductivity of a normal metal, $k_e = C_e v_e \Lambda_{e,e} / 3$,

where C_e is the electron specific heat per unit volume (Ziman, 1960). The electron specific heat of a normal metal is $C_e = \gamma T$, where γ is the Sommerfeld parameter. Based on these relations and the simplified form of Eq. (8), the temperature dependence of the electron mean free path limited by phonon scattering is

$$\frac{\Lambda_{e,s}(T)}{\Lambda_{e,s}(T_c)} = \frac{T_c^3 J_5 \left(\frac{\theta}{T_c} \right)}{T^3 J_5 \left(\frac{\theta}{T} \right)} \quad (9)$$

The Sommerfeld parameter is given by (Ziman, 1960)

$$\gamma = \frac{\pi^2 k_B^2 n}{m^* v_e^2} \quad (10)$$

This expression yields $\gamma = 140 \text{ J m}^{-3} \text{ K}^{-2}$ for $\text{YBa}_2\text{Cu}_3\text{O}_7$.

In kinetic theory, transport of thermal energy by a single carrier type through a reference plane depends on the flux of carriers through that plane from one side, Φ , and their exchange length with respect to the plane, L (Rohsenow and Choi, 1961). The exchange length is the average distance carriers passing through the plane have traveled normal to the plane since their previous collisions. In an isotropic medium in equilibrium these parameters are $\Phi = Nv/4$, where N is the number density of carriers and v is their average velocity, and $L = 2\Lambda/3$. In the present analysis, electron conduction in the a - b plane of $\text{YBa}_2\text{Cu}_3\text{O}_7$ is determined based on these quantities.

The electrons in a superconductor are modeled as existing in two states, the superconducting state and the normal state. The electron entropy per unit volume, S , is assumed to be a function of the entropy per unit volume the electrons would possess if they were all normal, S_n , and the fraction of the electrons that are in the normal state, X . Although the parameters X and S_n are independent in nonequilibrium states, in stable equilibrium states they are related through temperature, $X = X(T)$ and $S_n = S_n(T)$. An electron passing through a plane is assumed to contribute to the quantity S_n as if it were part of an electron gas in equilibrium at the temperature of its previous collision. The superconductor coherence length ξ is the distance over which a disturbance in the number density of superconducting electrons propagates. In the a - b plane of $\text{YBa}_2\text{Cu}_3\text{O}_7$, the superconductor coherence length is approximately 1.2 nm (Salamon, 1989). It will be shown in the present analysis that the a - b plane electron mean free path varies between $\Lambda_e = 6.3 \text{ nm}$ at zero temperature and $\Lambda_e = 4.5 \text{ nm}$ at the critical temperature in this material. This indicates that the distance over which parameter X is free to vary, ξ , is somewhat smaller than the mean free path, Λ_e . To account for this, the electrons traveling in either direction through a reference plane are assumed to exhibit a parameter X characteristic of the local temperature at the reference plane.

To determine the electron thermal conductivity in the superconducting state, the flux of thermal energy through the x - z plane located by $y = 0$ is calculated based on a temperature field $T(y)$ with $T(y = 0) = T_0$. The entropy flux through the plane due to electrons traveling in the positive y direction is $J_S^+ = \Phi S^+/n = v_e S^+/4$, where S^+ is the entropy per unit volume of these electrons as they reach the plane. Similarly, the entropy flux due to electrons traveling in the negative y direction is $J_S^- = v_e S^-/4$, where S^- is the entropy per unit volume of those electrons as they reach the plane. The average of the y coordinates of the locations of the previous collisions of carriers traveling through the plane in the positive y location is $-L = -2\Lambda_e/3$. The average of the y coordinates of the locations of the previous collisions of carriers traveling through the plane in the negative y direction is $2\Lambda_e/3$.

The net flux of entropy resulting from electron transport is

$$J_S = \frac{v_e}{4} [S^+ - S^-] = \frac{v_e}{4} \left[S \left(S_n \left(T \left(y = -\frac{2\Lambda_e}{3} \right) \right), X(T_0) \right) - S \left(S_n \left(T \left(y = \frac{2\Lambda_e}{3} \right) \right), X(T_0) \right) \right] \quad (11)$$

The temperature of the solid at points near the plane $y = 0$ is evaluated using Taylor's theorem for the function $T(y)$. Terms containing derivatives of second and higher order are neglected. Similarly, Taylor expansions yield the variation of $S = S(S_n, X)$ with respect to S_n and X about $S_n = S_n(T_0)$ and $X = X(T_0)$, and the variation of $S_n(T)$ and $X(T)$ with respect to T about $T = T_0$. Equation (11) becomes

$$J_S = \frac{v_e}{4} \left(-\frac{2}{3} \Lambda_e \right) \left(\frac{\partial S(S_n, X)}{\partial S_n} \right)_{y=0} \left(\frac{\partial S_n(T)}{\partial T} \right)_{y=0} \left(\frac{\partial T}{\partial y} \right)_{y=0} - \frac{v_e}{4} \left(\frac{2}{3} \Lambda_e \right) \left(\frac{\partial S(S_n, X)}{\partial S_n} \right)_{y=0} \left(\frac{\partial S_n(T)}{\partial T} \right)_{y=0} \left(\frac{\partial T}{\partial y} \right)_{y=0} = -\frac{v_e}{3} \Lambda_e \left(\frac{\partial S(S_n, X)}{\partial S_n} \right)_{y=0} \left(\frac{\partial S_n(T)}{\partial T} \right)_{y=0} \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (12)$$

The two-fluid model of electrons in the superconducting state (Gorter and Casimir, 1934; Shoenberg, 1952) provides the results $S(S_n, X) = S_n X^{1/2}$ and $X = (T/T_c)^4$ in equilibrium states at temperatures below T_c . In the present analysis the former relation is assumed to hold also in nonequilibrium states. Based on the Fermi free-electron model, the entropy of a normal electron gas is $S_n(T) = \gamma T$. The flux of thermal energy J_E is assumed to be the product of the temperature at the reference plane and the entropy flux,

$$J_E = T_0 J_S = -\frac{v_e}{3} \Lambda_e \gamma \frac{T_0^3}{T_c^2} \frac{\partial T}{\partial y} \quad (13)$$

The electron thermal conductivity at a temperature of $T = T_0$ is

$$k_e = \frac{J_E}{-\frac{\partial T}{\partial y}} = \frac{1}{3} \gamma \frac{T^3}{T_c^2} v_e \Lambda_e \quad (14)$$

Because the electron mean free path limited by defect scattering is independent of temperature, the normalized electron thermal conductivity limited by defect scattering is

$$\frac{k_{e,d}(T)}{k_{e,d}(T_c)} = \left(\frac{T}{T_c} \right)^3 \quad (15)$$

In Fig. 3, this simple result for the defect-limited electron thermal conductivity of superconductors is compared to that obtained by Geilikman and Kresin (1974) by solving the Boltz-

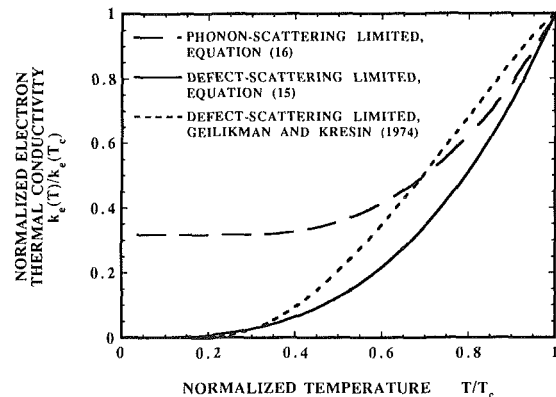


Fig. 3 Electron thermal conductivity in the superconducting state

mann equation. The two results agree within 35 percent. Equations (9) and (14) yield the electron thermal conductivity limited by phonon scattering,

$$\frac{k_{e,s}(T)}{k_{e,s}(T_c)} = \frac{J_5\left(\frac{\theta}{T_c}\right)}{J_5\left(\frac{\theta}{T}\right)}. \quad (16)$$

This result is also shown in Fig. 3. The phonon-limited electron thermal conductivity drops more gradually with decreasing temperature than its defect-limited counterpart because the phonon number density decreases with decreasing temperature. The phonon-limited electron thermal conductivity is finite at low temperatures, due to the competing effects of a phonon number density and a normal-state electron number density, which both decrease with decreasing temperature. The low-temperature limit of $k_{e,s}$ cannot be observed experimentally because defect scattering limits the electron thermal conductivity.

The electron thermal conductivity above the critical temperature $k_e(T_c)$ is calculated from the a - b plane electrical resistivity ρ above the critical temperature using the Wiedemann-Franz law,

$$k_e = \eta \frac{L_0 T}{\rho}, \quad 0 < \eta \leq 1, \quad (17)$$

where η is predicted by (Wilson, 1953)

$$\eta = \frac{\frac{\rho_d}{4\rho_\theta} + \zeta\left(\frac{T}{\theta}\right)}{\frac{\rho_d}{4\rho_\theta} + \chi\left(\frac{T}{\theta}, n, N_a\right)}. \quad (18)$$

The functions ζ and χ are defined in Eqs. (5) and (8), respectively. The coefficient η approaches unity at temperatures either large or small compared to the Debye temperature θ , and achieves a minimum value at a temperature less than θ . The larger the magnitude of the defect resistivity, indicated by the ratio ρ_d/ρ_θ , the closer is η to unity at temperatures comparable to θ . When scattering on defects dominates over scattering on phonons, Eq. (17) may be used with $\eta = 1$. The values of the parameters ρ_d , n , and N_a are given for $\text{YBa}_2\text{Cu}_3\text{O}_7$ in this section and $\theta = 470$ K is calculated in the following section. The parameter ρ_θ is determined to be $3.9 \mu\Omega\text{m}$ by fitting Eqs. (4) and (5) to the electrical resistivity data of Cohn et al. (1992b). The coefficient η is larger than 0.91 at all temperatures, and the deviation of η from unity is neglected in the present analysis. The electron mean free path Λ_e is calculated at temperatures greater than the critical temperature using Eq. (17) and the relations $k_e = C_e v_e \Lambda_e / 3$ and $C_e = \gamma T$. At lower temperatures, it is determined using Eqs. (9), (2), and (1). The electron thermal conductivity in the superconducting state is calculated using Eq. (14).

Phonon Conductivity and Mean Free Path

The a - b plane thermal conductivity k is the sum of the electron component k_e and the phonon component k_s (Wilson, 1953)

$$k = k_e + k_s. \quad (19)$$

The phonon thermal conductivity is obtained by subtracting the predicted electron contribution from the experimentally measured a - b plane thermal conductivity. The phonon mean free path Λ_s is calculated from the phonon thermal conductivity using the relation from kinetic theory for the phonon thermal conductivity (Ziman, 1960),

$$k_s = \frac{1}{3} C_s v_s \Lambda_s, \quad (20)$$

where C_s is the specific heat per unit volume of phonons. The velocity v_s is the average speed of sound, which can be calculated from the measured velocity of longitudinal sound waves v_l and transverse sound waves v_t using $v_s^3 \doteq 3/(2/v_l^3 + 1/v_t^3)$. It is likely that the average sound velocity is anisotropic in single-crystal $\text{YBa}_2\text{Cu}_3\text{O}_7$, i.e., that it has a value along the c axis different from that in the a - b plane. However, this anisotropy has not been experimentally demonstrated because the small size of available single crystals has prevented the accurate measurement of the absolute magnitude of the velocity of sound. Saint-Paul et al. (1988) showed that the transverse and longitudinal sound velocities vary with temperature by less than 5 percent of their absolute magnitude from 40 K to 160 K. The present analysis uses an average sound velocity, which is independent of temperature between 0 K and 120 K. The average sound velocity is obtained from measurements of the Debye temperature θ using (Kittel, 1986)

$$v_s = \frac{2\pi k_B \theta}{h_p (6\pi^2 N_a)^{1/3}}, \quad (21)$$

where $N_a = 7.5 \times 10^{28} \text{ m}^{-3}$ is the number density of atoms in $\text{YBa}_2\text{Cu}_3\text{O}_7$. The Debye temperature is determined from experimental data for the phonon specific heat using (Ziman, 1960)

$$C_s = 9N_a k_B \left(\frac{T}{\theta}\right)^3 J_4\left(\frac{\theta}{T}\right), \quad (22)$$

where the Debye integral $J_p(q)$ is defined in Eq. (6).

Beckman et al. (1987) showed that 97 percent of the specific heat of bulk $\text{YBa}_2\text{Cu}_3\text{O}_7$ is due to lattice vibrations. In the present analysis, the phonon specific heat of single-crystal $\text{YBa}_2\text{Cu}_3\text{O}_7$ is calculated from measurements of the total specific heat of polycrystalline bulk samples. The specific heat of the porous bulk samples is multiplied by the ratio of the mass density of a perfect single crystal to the measured mass density of the bulk samples. From 40 K to 120 K the data of Lang et al. (1988) are used, and from 10 K to 40 K the data of Ferreira et al. (1988) are used. Since the measurements at 40 K differed by 17 percent, the low-temperature data are multiplied by the ratio $C(40 \text{ K, Lang et al., 1988})/C(40 \text{ K, Ferreira et al., 1988})$ to provide a smooth transition. The specific heat C_s at 100 K is $1170 \text{ kJ m}^{-3} \text{ K}^{-1}$. The Debye temperature calculated using Eq. (22) varies from $\theta = 390$ K at a temperature of 10 K to $\theta = 520$ K at a temperature of 120 K. For the present analysis, the Debye temperature $\theta = 470$ K at a temperature of 90 K is employed, from which Eq. (21) yields an average sound velocity of $v_s = 3800 \text{ m s}^{-1}$. This compares well with the values of $v_l = 4000 \text{ m s}^{-1}$ and $v_t = 2500 \text{ m s}^{-1}$ measured by Lang et al. (1988) in sintered samples.

Size Effect

The reduction of the conductivity of a heat carrier along a film resulting from boundary scattering is calculated from the ratio of the film thickness to the bulk mean free path, $\delta = d/\Lambda$, using the approximate theory of Flik and Tien (1990c). Their result differs by less than 20 percent from the solution to the Boltzmann equation given by Sondheimer (1952) for electrons. For $\delta > 1$,

$$\frac{k_{\text{eff}}}{k} = 1 - \frac{2}{3\pi\delta}, \quad (23)$$

where k_{eff} is the thermal conductivity along the thin film and k is the bulk conductivity. For $\delta < 1$,

$$\frac{k_{\text{eff}}}{k} = 1 - \frac{2(1-Z^3)}{3\pi\delta} + \frac{2\delta}{\pi} \ln \frac{1+\delta+Z}{1+\delta-Z} - \frac{2}{\pi} \arccos \delta, \quad (24)$$

where $Z = (1 - \delta^2)^{1/2}$. Use of the bulk value of the thermal conductivity results in an error of less than 5 percent in the

predicted heat flux if the film thickness satisfies $d > 4.5\lambda$. Equations (23) and (24) are applied to the electron and phonon systems separately, yielding the effective conductivity of each carrier type from its bulk conductivity and mean free path, properties determined in the previous sections. The thermal conductivity of the film is equal to the sum of the reduced electron and phonon components.

The use of Eqs. (23) and (24) neglects the anisotropy of epitaxial $\text{YBa}_2\text{Cu}_3\text{O}_7$. These equations were derived for an isotropic material in which the ends of the free paths originating from a point fall on the surface of a sphere in the absence of boundary scattering. An estimate of the effect of phonon-boundary scattering on conduction in an anisotropic film indicates the error resulting from this simplification. The ratio of the phonon mean free path in the a - b plane to that along the c axis is approximately the square root of the ratio of the thermal conductivities, $\Lambda_{s,a-b}/\Lambda_{s,c} = (k_{a-b}/k_c)^{1/2}$ (Flik and Tien, 1990c). The conductivity ratio is $k_{a-b}/k_c \sim 5$ in $\text{YBa}_2\text{Cu}_3\text{O}_7$ (Hagen et al., 1989). The ends of the free paths originating from a point are assumed to fall on the surface of an ellipsoid whose axis of rotational symmetry is normal to the a - b plane, whose height normal to the a - b plane is $2\Lambda_{s,c}$, and whose cross section in the a - b plane containing the origination point is a circle of radius $\Lambda_{s,a-b}$. The effect of boundary scattering is calculated using the technique of Flik and Tien (1990c) with $\Lambda_{s,a-b} = \Lambda_{s,\text{isotropic}}$. Neglecting the anisotropy results in an overestimation of the size effect. The error is 36 percent for $\delta \ll 1$, 12 percent for $\delta = 1$, and approaches zero for $\delta \gg 1$.

The large anisotropy of the electrical resistivity, $\rho_c/\rho_{a-b} \sim 250$ above T_c in $\text{YBa}_2\text{Cu}_3\text{O}_7$ (Hagen et al., 1988) indicates that one or more of the quantities m^* , n , or the electron scattering time $\tau = \Lambda_e/v_e$ is less favorable for transport parallel to the c axis than for a - b plane transport. If either n or τ is responsible, the fraction of electron free paths that terminate at the film boundaries is smaller than that in an isotropic film, and the present analysis overpredicts the size effect on the electron thermal conduction. The electron contribution to the total conductivity is smaller than that of phonons, particularly at temperatures below the critical temperature, reducing the importance of this error. It will be shown that the size effect on the phonons dominates over the size effect on the electrons, further reducing the importance of this error.

Equations (23) and (24) are valid if phonons and electrons are reflected diffusely at the film surface. Diffuse reflection occurs if the carrier wavelength is of the order of or smaller than the standard deviation of the surface profile multiplied by 4π (Ziman, 1960). This is almost certainly satisfied by electrons in $\text{YBa}_2\text{Cu}_3\text{O}_7$, which have a wavelength of $2\hbar p/(v_e m^*) \sim 2$ nm. The average phonon wavelength is approximately $\hbar p v_s/k_B T$, which in $\text{YBa}_2\text{Cu}_3\text{O}_7$ at 25 K is 7 nm. In order for diffuse reflection to prevail at this temperature, roughness features of $6 \text{ \AA}$ must be present at the boundaries of the film. Majumdar and Tien (1990) analyzed the condition for diffuse reflection using a fractal description of the surface. If the fractal surface parameters can be determined from experiments, a limiting temperature can be determined above which most reflections are diffuse. Chen and Tien (1992) showed that the diffuse transmission of carriers through the boundaries does not influence the effect of boundary scattering on thermal conduction along a film. At low temperatures, where longer phonon wavelengths may decrease the fraction of phonons that are diffusely transmitted, the fraction of incident phonons transmitted through an interface is reduced by the acoustic mismatch of the two materials (Swartz and Pohl, 1989).

Summary of Technique

The technique developed in the present analysis for calculating the effective thermal conductivity in epitaxial high- T_c

superconducting films is appropriate for a superconductor in which both electrons and phonons contribute significantly to thermal conduction. Application of the Wiedemann-Franz law using normal-state electrical resistivity data has indicated that this is the case for the a - b plane of single crystal $\text{YBa}_2\text{Cu}_3\text{O}_7$ and $\text{Bi}_2\text{Sr}_2\text{Ca}_1\text{Cu}_2\text{O}_8$, as well as for conduction in polycrystalline $\text{EuBa}_2\text{Cu}_3\text{O}_{7-x}$ (Izbizky et al., 1988) and $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ with $x > 0.3$ (Sera et al., 1990).

Requirements

- 1 The electrical resistivity $\rho(T)$, $T_c < T < 300$ K, and the thermal conductivity $k(T)$, $20 \text{ K} < T < 300$ K of a large specimen.
- 2 The specific heat of the material, which is dominated by phonons, $C_s(T)$, $20 \text{ K} < T < 300$ K.
- 3 The effective mass m^* and number density n of electrons.

Procedure

- 1 Equations from the Sommerfeld free-electron model modified by an effective electron mass yield the properties of electrons. Calculate v_e using Eq. (3) and γ using Eq. (10).
- 2 Equations from the Debye model yield the properties of phonons. Calculate θ from the data $C_s(T)$ using Eq. (22), and calculate v_s using Eq. (21).
- 3 The Bloch formula for the phonon electrical resistivity and Matthiessen's rule yield the defect electrical resistivity. Choose as T_R the upper temperature of the superconducting to normal-state transition as deduced from $\rho(T)$. Calculate ρ_d from the data $\rho(T)$ using Eq. (7).
- 4 The kinetic-theory formulation of the electrical resistivity yields the electron mean free path limited by defect scattering. Calculate $\Lambda_{e,d}$, which is independent of temperature, from ρ_d using Eq. (2).
- 5 The Wiedemann-Franz law yields the electron thermal conductivity at the critical temperature. Calculate $k_e(T_c)$ from $\rho(T_R)$ and Eq. (17).
- 6 The kinetic-theory formulation of the thermal conductivity yields the electron mean free path at the critical temperature. Calculate $\Lambda_e(T_c)$ using the normal-state relation, $k_e(T_c) = C_e(T_c)v_e\Lambda_e(T_c)/3$, where $C_e(T_c) = \gamma T_c$.
- 7 Matthiessen's rule yields the electron mean free path limited by phonon scattering at the critical temperature. Calculate $\Lambda_{e,s}(T_c)$ from $\Lambda_e(T_c)$ and $\Lambda_{e,d}(T_c)$ using Eq. (1).
- 8 The Wilson formula for the electron thermal conductivity limited by phonon scattering yields the electron mean free path limited by phonon scattering in the superconducting state. Calculate $\Lambda_{e,s}(T)$, $0 < T < T_c$, from $\Lambda_{e,s}(T_c)$ using Eq. (9).
- 9 Matthiessen's rule yields the electron mean free path in the superconducting state. Calculate $\Lambda_e(T)$, $0 < T < T_c$, using $\Lambda_{e,s}(T)$ and $\Lambda_{e,d}$ in Eq. (1).
- 10 The kinetic method and the expression for the electron entropy given by the two-fluid model of electrons in superconductors yield the electron thermal conductivity in the superconducting state. Calculate $k_e(T)$, $0 < T < T_c$, from $\Lambda_e(T)$ using Eq. (14).
- 11 The assumption that the electron and phonon conductivities sum yields the phonon conductivity in the superconducting state. Calculate $k_s(T)$ from $k_e(T)$ and the data $k(T)$ using Eq. (19).
- 12 The kinetic-theory formulation of the thermal conductivity yields the phonon mean free path. Calculate $\Lambda_s(T)$ from $k_s(T)$ and the data $C_s(T)$ using Eq. (20).
- 13 Equations that account for the size effect in thin films yield the effective electron and phonon conductivities reduced by boundary scattering. Given a film of thickness d , calculate $k_{e,\text{eff}}(T)$ from $k_e(T)$ and $\Lambda_e(T)$ using Eqs. (23) and (24). Calculate $k_{s,\text{eff}}(T)$ from $k_s(T)$ and $\Lambda_s(T)$ using Eqs. (23) and (24).
- 14 Calculate $k_{\text{eff}}(T)$ by summing $k_{e,\text{eff}}(T)$ and $k_{s,\text{eff}}(T)$.

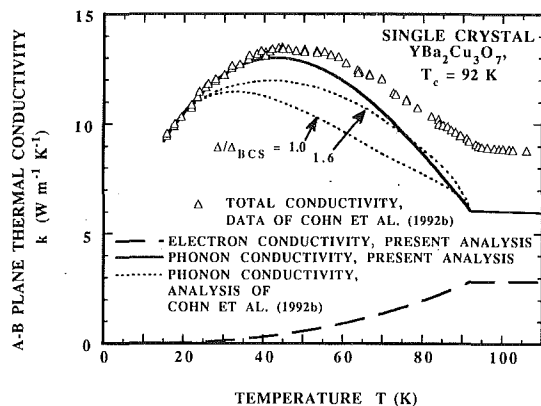


Fig. 4 Data for the a - b plane thermal conductivity of a $\text{YBa}_2\text{Cu}_3\text{O}_7$ crystal in the absence of boundary scattering, and predictions of the electron and phonon contributions

Results and Discussion

In this section, the technique developed in the previous sections for calculating the effective conductivity along high- T_c superconducting films is applied to the a - b plane of epitaxial $\text{YBa}_2\text{Cu}_3\text{O}_7$ films. The thermal conductivity and electrical resistivity data employed to calculate the electron and phonon conductivities and mean free paths in the absence of the size effect are those of crystal CR3 of Cohn et al. (1992b), the only single crystal for which both the electrical resistivity and the thermal conductivity have been reported as functions of temperature. The electron and phonon contributions to the a - b plane thermal conductivity of the crystal are shown in Fig. 4. The data exhibit the broad peak in the conductivity typical of high- T_c superconductors, which results from the phonon contribution to the conductivity. The phonon conductivity increases with decreasing temperature because of the decreasing number density of normal-state electrons, which are important scatterers of phonons. Superconducting electrons do not exchange energy or momentum with the lattice and do not scatter phonons. At temperatures below about 50 K, this enhancement of the phonon conductivity is offset by the decreasing number density of phonons with decreasing temperature. The electron contribution to the conductivity decreases with decreasing temperature because of the diminishing number density of normal-state electrons. Superconducting electrons carry no entropy and do not contribute to heat transport. The electron thermal conductivity is less than 10 percent of the total conductivity below 60 K.

Cohn et al. (1992b) fitted the phonon-conductivity formulation of Tewordt and Wölkhausen (1990) to the data at low temperatures, where the electron contribution is negligible, and to the phonon contribution determined using Eqs. (17) and (19) above the critical temperature. They employed different values of the superconductor energy gap compared to that of Bardeen et al. (1957), $\Delta/\Delta_{\text{BCS}}$, a ratio whose correct value is not known for $\text{YBa}_2\text{Cu}_3\text{O}_7$. The phonon conductivity predicted in the present analysis agrees within 10 percent with that of Cohn et al. (1992b) with $\Delta/\Delta_{\text{BCS}} = 1.6$. The present analysis determines that the electron conductivity is limited by defect scattering at temperatures up to the critical temperature, i.e., the second term on the right of Eq. (1) is always larger than the other terms. Each phonon conductivity prediction of Cohn et al. (1992b), when subtracted from the data, yields an electron conductivity that has a local maximum near 55 K. This is not consistent with the present analysis, nor with the electron conductivity limited by defect scattering of Geilikman and Kresin (1974).

Figure 5 shows the temperature dependence of the electron and phonon thermal conduction mean free paths in the absence of boundary scattering. The close similarity of the electron

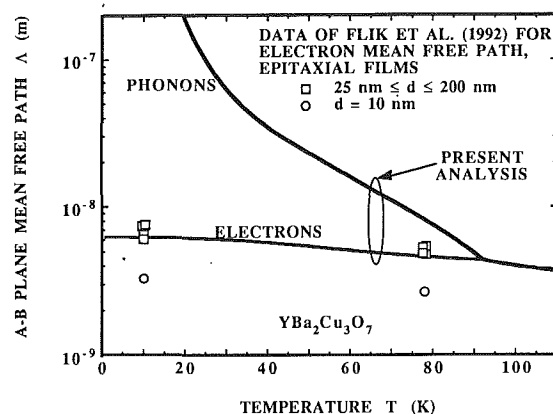


Fig. 5 Electron and phonon mean free paths in the a - b plane of $\text{YBa}_2\text{Cu}_3\text{O}_7$

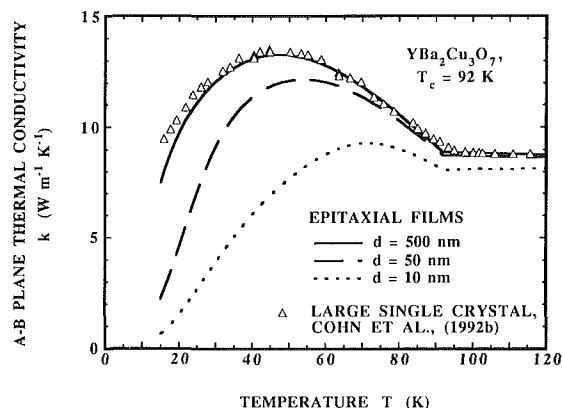


Fig. 6 Size effect on the a - b plane thermal conductivity of epitaxial $\text{YBa}_2\text{Cu}_3\text{O}_7$ films

and phonon mean free paths above the critical temperature is coincidental and is not reproduced by applying the technique of the present analysis to another crystal. The data are those of Flik et al. (1991), who developed a technique for fitting the experimentally measured infrared reflectance of $\text{YBa}_2\text{Cu}_3\text{O}_7$ films in the superconducting state, which yielded the electron mean free path Λ_e . The technique was applied to five films of thicknesses 10 nm, 25 nm, 50 nm, 100 nm, and 200 nm. At both 10 K and 78 K, the mean free path data are in excellent agreement with the predictions of the present analysis, with the exception of those for the 10 nm film, which indicate relatively small mean free paths. The different scattering rate in the 10 nm film may have resulted either from a different microstructure or from the increased importance of boundary scattering, since the thickness of the film was comparable to the electron mean free path (Flik et al., 1991).

The electron mean free path increases gradually with decreasing temperature because of the decreasing number density of phonons. The electron mean free path is limited by defect scattering, and therefore depends weakly on temperature and has a magnitude that depends on the microstructure of the sample. The phonon mean free path increases with decreasing temperature below the critical temperature because of the decreasing number density of normal-state electrons, which scatter phonons. The number density of normal-state electrons is constant above the critical temperature, resulting in a discontinuity in the temperature derivative of the phonon mean free path at the critical temperature. Phonons of wavelength large compared to the dimensions of point defects, dislocations, and impurities scatter less frequently on these defects than phonons of wavelength comparable to or smaller than the dimensions of these defects. The average phonon wavelength is inversely

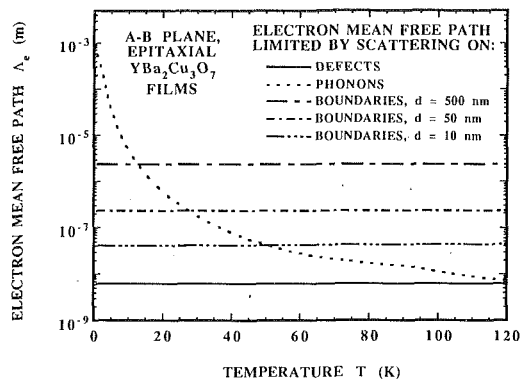


Fig. 7 Electron mean free paths for a - b plane thermal conduction in epitaxial $\text{YBa}_2\text{Cu}_3\text{O}_7$ films limited by scattering on defects, phonons, and the film boundaries

proportional to temperature, and the phonon mean free path increases quickly with decreasing temperature at low temperatures because of a decreasing rate of scattering on defects.

The predicted a - b plane thermal conductivities of epitaxial $\text{YBa}_2\text{Cu}_3\text{O}_7$ films of varying thickness d are shown in Fig. 6. The data are those for a large crystal and are included as the prediction for the limit of very thick films. Boundary scattering is negligible at temperatures above 20 K for a film thickness $d = 500$ nm. The thermal conductivity of the 50 nm film exhibits a significant size effect for temperatures below 55 K. Since the electron mean free path is much smaller than the film thickness in the entire temperature range, this size effect is due phonon-boundary scattering. The 10 nm film is of thickness comparable with the electron mean free path, which varies slowly with temperature, and boundary scattering is important in the entire temperature range shown. Because of the small relative magnitude of the electron thermal conductivity at low temperatures and the low electron mean free path, a significant size effect on the electron thermal conduction is confined to temperatures greater than 60 K and films of thickness less than 30 nm.

The components of the electron mean free path given in Eq. (1) are shown in Fig. 7. The electron mean free path limited by boundary scattering $\Lambda_{e,b}$ is calculated from Eq. (1) and the theoretical results for $\Lambda_{e,s}$ and $\Lambda_{e,d}$. The small relative magnitude of $\Lambda_{e,d}$ indicates that defect scattering is the dominant mechanism limiting electron conduction for each film thickness below 120 K.

Conclusions

The determination of the effective conductivity along epitaxial high- T_c superconducting films must consider the size effect on electrons and phonons separately only if the electron thermal conductivity is appreciable. This work shows that in the a - b plane of $\text{YBa}_2\text{Cu}_3\text{O}_7$ the phonon conductivity dominates over the electron conductivity at temperatures below 60 K, where the size effect in films is of the greatest importance. Below 60 K, the size effect may be calculated considering only phonon conduction.

This work calculates for the first time the mean free path of normal-state electrons in the superconducting state of $\text{YBa}_2\text{Cu}_3\text{O}_7$, and the result is in close agreement with data from infrared spectroscopy at 10 K and 78 K. This agreement is a strong endorsement of the simplifications that make the present analysis possible. The electron mean free path is limited by scattering on defects and increases gradually with decreasing temperature to 63 Å at 10 K.

The predictions of the present analysis should be compared to experimental data for the thermal conductivity along epitaxial high- T_c superconducting films. There is an urgent need

for experimental techniques that measure the thermal conductivity in epitaxial thin films such as those considered in this study.

Acknowledgments

K. E. G. acknowledges the support of the Office of Naval Research through an academic fellowship. Professor M. S. Dresselhaus provided valuable comments. The authors are indebted to Dr. S. A. Wolf for providing a preprint of his manuscript (Cohn et al., 1992b).

References

- Bardeen, J., Cooper, L. N., and Schrieffer, J. R., 1957, "Theory of Superconductivity," *Phys. Rev.*, Vol. 108, pp. 1175-1204.
- Bardeen, J., Rickayzen, G., and Tewordt, L., 1959, "Theory of the Thermal Conductivity of Superconductors," *Phys. Rev.*, Vol. 113, pp. 982-994.
- Beckman, O., Lundgren, P., Nordblad, P., Sandlund, L., Svendlindh, P., Lundström, T., and Rundqvist, S., 1987, "Specific Heat and Magnetic Susceptibility of Single-Phase $\text{YBa}_2\text{Cu}_3\text{O}_7$," *Phys. Lett. A*, Vol. 125, pp. 425-428.
- Bloch, F., 1930, "Zum Elektrischen Widerstandsgesetz bei tiefen Temperaturen," *Z. Phys.*, Vol. 59, pp. 209-214.
- Chen, G., and Tien, C. L., 1992, "Thermal Conductivity of Quantum Well Structures," AIAA Paper No. 92-0707.
- Cohn, J. L., Wolf, S. A., and Vanderah, T. A., 1992a, "Evidence for Strong Electron-Phonon Coupling in the Thermal Conductivity of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$," *Phys. Rev. B*, Vol. 45, pp. 511-514.
- Cohn, J. L., Wolf, S. A., Vanderah, T. A., Selvamanickam, V., and Salama, K., 1992b, "Lattice Thermal Conductivity of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$," *Physica C*, Vol. 192, pp. 435-442.
- Crommie, M. F., and Zettl, A., 1990, "Thermal Conductivity of Single-Crystal Bi-Sr-Ca-Cu-O," *Phys. Rev. B*, Vol. 41, pp. 10978-10982.
- Ferreira, J. M., Lee, B. W., Dalichaouch, Y., Torikachvili, M. S., and Maple, M. B., 1988, "Low-Temperature Specific Heat of the High- T_c Superconductors $\text{La}_{1-x}\text{Sr}_x\text{CuO}_{4-\delta}$ and $\text{R}\text{Ba}_2\text{Cu}_3\text{O}_{7-\delta}$ ($R = \text{Y, Eu, Ho, Tm, and Yb}$)," *Phys. Rev. B*, Vol. 37, pp. 1580-1586.
- Fiory, A. T., Hebard, A. F., Eick, R. H., Mankiewich, P. M., Howard, R. E., and O'Malley, M. L., 1990, "Metallic and Superconducting Surfaces of $\text{YBa}_2\text{Cu}_3\text{O}_7$ Probed by Electrostatic Charge Modulation of Epitaxial Films," *Phys. Rev. Lett.*, Vol. 65, pp. 3441-3444.
- Flik, M. I., and Hijikata, K., 1990, "Approximate Thermal Packaging Limit for Hybrid Superconductor-Semiconductor Electronic Circuits," *Heat Transfer 1990*, G. Hetsroni, ed., Hemisphere, New York, Vol. 2, pp. 319-324.
- Flik, M. I., Phelan, P. E., and Tien, C. L., 1990, "Thermal Model for the Bolometric Response of High- T_c Superconducting Films to Optical Pulses," *Cryogenics*, Vol. 30, pp. 1118-1128.
- Flik, M. I., and Tien, C. L., 1990a, "Intrinsic Thermal Stability of Anisotropic Thin-Film Superconductors," *ASME JOURNAL OF HEAT TRANSFER*, Vol. 112, pp. 10-15.
- Flik, M. I., and Tien, C. L., 1990b, "Intrinsic Thermal Stability for Scanning Electron Microscopy of Thin-Film Superconductors," *J. Appl. Phys.*, Vol. 67, pp. 362-370.
- Flik, M. I., and Tien, C. L., 1990c, "Size Effect on the Thermal Conductivity of High- T_c Thin-Film Superconductors," *ASME JOURNAL OF HEAT TRANSFER*, Vol. 112, pp. 872-881.
- Flik, M. I., Zhang, Z. Z., and Goodson, K. E., 1992, "The Electron Scattering Rate in Epitaxial $\text{YBa}_2\text{Cu}_3\text{O}_7$ Superconducting Films," *Phys. Rev. B*, Vol. 46, pp. 5606-5614.
- Flik, M. I., and Goodson, K. E., 1992, "Thermal Analysis of Electron-Beam Absorption in Low-Temperature Superconducting Films," *ASME JOURNAL OF HEAT TRANSFER*, Vol. 114, pp. 264-270.
- Geilikman, B. T., and Kresin, V. Z., 1974, *Kinetic and Nonsteady-State Effects in Superconductors*, Wiley, New York, pp. 5-7, 13-15.
- Gorter, C. J., and Casimir, H., 1934, "On Supraconductivity I," *Physica*, Vol. 1, pp. 306-320.
- Hagen, S. J., Jing, T. W., Wang, Z. Z., Horvath, J., and Ong, N. P., 1988, "Out-of-Plane Conductivity in Single-Crystal $\text{YBa}_2\text{Cu}_3\text{O}_7$," *Phys. Rev. B*, Vol. 37, pp. 7928-7931.
- Hagen, S. J., Wang, Z. Z., and Ong, N. P., 1989, "Anisotropy of the Thermal Conductivity of $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$," *Phys. Rev. B*, Vol. 40, pp. 9389-9392.
- Izizky, M. A., Regueiro, M. N., Esquinazi, P., and Fainstein, C., 1988, "Thermal Conductivity and Superconductivity in $\text{EuBa}_2\text{Cu}_3\text{O}_{7-x}$," *Phys. Rev. B*, Vol. 38, pp. 9220-9223.
- Kittel, C., 1986, *Introduction to Solid State Physics*, Wiley, New York, pp. 106, 127-130, 135-140, 264-267.
- Kumar, S., and Joshi, A., 1990, "Thermal Analysis of a Transition-Edge High T_c Superconductor Infrared Detector," *Radiation Heat Transfer: Fundamentals and Applications*, T. F. Smith et al., eds., ASME HTD-Vol. 137, pp. 173-180.
- Lang, M., Lechner, T., Riegel, S., Steglich, F., Weber, G., Kim, T. J., Lüthi, B., Wolf, B., Rietschel, H., and Wilhelm, M., 1988, "Thermal Expansion, Sound Velocities, Specific Heat and Pressure Derivative of T_c in $\text{YBa}_2\text{Cu}_3\text{O}_7$," *Z. Phys. B*, pp. 459-463.
- Lavine, A. S., and Bai, C., 1990, "An Analysis of Heat Transfer in Josephson Junction Devices," *ASME JOURNAL OF HEAT TRANSFER*, Vol. 113, pp. 535-543.

- Majumdar, A., and Tien, C. L., 1990, "Reflection of Radiation by Rough Fractal Surfaces," presented at the ASME/AIAA Thermophysics and Heat Transfer Conference, Seattle, WA, June 13–20.
- Mannhart, J., Gross, R., Hipler, K., Huebener, R. P., Tsuei, C. C., Dimos, D., and Chaudari, P., 1989, "Spatially Resolved Observation of Supercurrents Across Grain Boundaries in YBaCuO Films," *Science*, Vol. 245, pp. 839–840.
- Peacor, S. D., Cohn, J. L., and Uher, C., 1991, "Effect of Magnetic Field on Thermal Conductivity of YBa₂Cu₃O_{7-δ} Single Crystals," *Phys. Rev. B*, Vol. 43, pp. 8721–8724.
- Penney, T., von Molnár, S., Kaiser, D., Holtzberg, F., and Kleinsasser, A. W., 1988, "Strongly Anisotropic Electrical Properties of Single-Crystal YBa₂Cu₃O_{7-x}," *Phys. Rev. B*, Vol. 38, pp. 2918–2921.
- Rohsenow, W. M., and Choi, H. Y., 1961, *Heat, Mass, and Momentum Transfer*, Prentice-Hall, Englewood Cliffs, NJ, pp. 486–501.
- Saint-Paul, M., Tholence, J. L., and Monceau, P., 1988, "Ultrasound Study of YBa₂Cu₃O_{7-δ} Single Crystals," *Solid State Comm.*, Vol. 66, pp. 641–643.
- Salamon, M. B., 1989, "Thermodynamic Properties, Fluctuations, and Anisotropy of High Temperature Superconductors," in: *Physical Properties of High Temperature Superconductors I*, D. M. Ginsberg, ed., World Scientific Publishing Co., Teaneck, NJ, pp. 39–70.
- Sera, M., Shamoto, S., Sato, M., Watanabe, I., Nakashima, S., and Kumagai, K., 1990, "Thermal Conductivity of High-*T_c* Oxides," *Solid State Comm.*, Vol. 74, pp. 951–956.
- Shoenberg, D., 1952, *Superconductivity*, Cambridge University Press, New York.
- Sondheimer, E. H., 1952, "The Mean Free Path of Electrons in Metals," *Advances in Physics*, Vol. 1, pp. 1–42.
- Stormer, H. L., Levi, A. F. J., Baldwin, K. W., Anzlowar, M., and Boebinger, G. S., 1988, "Normal-State Transport Parameters of Epitaxial Thin Films of YBa₂Cu₃O_{7-δ}," *Phys. Rev. B*, Vol. 38, pp. 2472–2476.
- Swartz, E. T., and Pohl, R. O., 1989, "Thermal Boundary Resistance," *Rev. Mod. Phys.*, Vol. 61, pp. 605–668.
- Tewordt, L., and Wölkhäusen, T., 1989, "Theory of Thermal Conductivity of the Lattice for High-*T_c* Superconductors," *Solid State Comm.*, Vol. 70, pp. 839–844.
- Tewordt, L., and Wölkhäusen, T., 1990, "Theory of Phonon Thermal Conductivity for Strong-Coupling s- and d-Wave Pairing in High *T_c* Superconductors," *Solid State Comm.*, Vol. 75, pp. 515–519.
- Uher, C., 1990, "Thermal Conductivity of High-*T_c* Superconductors," *Journal of Superconductivity*, Vol. 3, pp. 337–389.
- Van Duzer, T., 1988, "Superconductor-Semiconductor Hybrid Devices, Circuits and Systems," *Cryogenics*, Vol. 28, pp. 527–531.
- Wilson, A. H., 1953, *The Theory of Metals*, Cambridge University Press, New York, pp. 285, 290.
- Ziman, J. M., 1960, *Electrons and Phonons*, Oxford University Press, Oxford, United Kingdom, pp. 105–107, 259, 257–261, 285–287, 357–367, 456–460.